4-6 Cramer's Rule

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- used determinants to solve systems of equations

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Use elimination to solve for x:

$-c(ax + by = e)$

$+a(cx + dy = f)$

$-acx - bcy = -ec$

$+acx + ady = +af$

$(ad - bc)y = af - ec$

$y = \dfrac{af - ec}{ad - bc}$

$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$\begin{vmatrix} a & e \\ c & f \end{vmatrix}$

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$$x = \frac{de - bf}{ad - bc}$$

$$ax + by = e$$
$$cx + dy = f$$

Denominator is:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = D$$

Write a determinant matrix for the numerator:

$$D_y = \begin{vmatrix} a & e \\ c & f \end{vmatrix}$$

$$D_x = \begin{vmatrix} e & b \\ f & d \end{vmatrix}$$

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Cramer's Rule:

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D}$$

Where D_x replaces the coefficients of x with the constants and D_y replaces the coefficients of y with the constants.

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ex:

x - 3y = 5

2x + 9y = -10

$x = \frac{D_x}{D}$

$D = \begin{vmatrix} 1 & -3 \\ 2 & 9 \end{vmatrix} = 15$

$x = \frac{15}{15} = 1$

$D_x = \begin{vmatrix} 5 & -3 \\ -10 & 9 \end{vmatrix} = 15$

$y = \frac{D_y}{D}$

$= \frac{-20}{15} = -\frac{4}{3}$

$D_y = \begin{vmatrix} 1 & 5 \\ 2 & -10 \end{vmatrix} = -20$

$\left(1, -\frac{4}{3}\right)$

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If D = 0

then, either

∞ # of solutions

OR

∅

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ex:
2x - y = 3
6x - 3y = -9

$(\quad)^{-3}$

$-6x + 3y = -9$
$6x - 3y = -9$
$0 \neq -18$

$D = \begin{vmatrix} 2 & -1 \\ 6 & -3 \end{vmatrix} = 0$

$D_x = \begin{vmatrix} 3 & -1 \\ -9 & -3 \end{vmatrix}$

$\varnothing$

$D_y = \begin{vmatrix} 2 & 3 \\ 6 & -9 \end{vmatrix}$

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3 Variables

Cramer's Rule:

$x = \frac{D_x}{D} \quad y = \frac{D_y}{D} \quad z = \frac{D_z}{D}$

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ex:
$4x + y - z = -2$
$x + 3y - 4z = 1$
$2x - y + 3z = 4$

Use calc.

$$D = \begin{vmatrix} 4 & 1 & -1 \\ 1 & 3 & -4 \\ 2 & -1 & 3 \end{vmatrix} = 16$$

$$D_x = \begin{vmatrix} -2 & 1 & -1 \\ 1 & 3 & -4 \\ 4 & -1 & 3 \end{vmatrix} = -16$$

$$D_y = \begin{vmatrix} 4 & -2 & -1 \\ 1 & 1 & -4 \\ 2 & 4 & 3 \end{vmatrix} = 96$$

$$D_z = \begin{vmatrix} 4 & 1 & -2 \\ 1 & 3 & 1 \\ 2 & -1 & 4 \end{vmatrix} = 64$$

$(-1, 6, 4)$

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HW

p 192-193

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